Conference

Structures Semester: Descriptive Set Theory & Dynamics

University of Warsaw
Faculty of Mathematics, Informatics and Mechanics

Banach Center
Institute of Mathematics PAS

Schedule

Monday

8:30-9:00	registration
9:00-9:15	opening
9:15-10:00	**Sławomir Solecki**, *Descriptive Set Theory and generic...*
10:00-10:20	coffee break
10:20-11:05	**Krzysztof Krupiński**, *Ramsey theory and topological...*
11:05-11:15	break
11:15-12:00	**Todor Tsankov**, *Maximal highly proximal flows of locally...*
12:00-14:00	lunch break
14:00-14:45	**Andrew Zucker**, *Ultracoproducts of G-flows*
14:45-15:05	coffee break
15:05-15:50	**Aleksandra Kwiatkowska**, *Dendroids via the projective...*
15:50-16:00	break
16:00-16:45	**Gábor Kun**, *When the measurable Hall theorem fails*
16:45-16:55	break
16:55-17:40	**Ádám Timár**, *Poisson matchings of optimal tail via...* (zoom)

Tuesday

8:30-9:15	registration
9:15-10:00	**Oleg Pikhurko**, *Circle Squaring with Pieces of Small...*
10:00-10:20	coffee break
10:20-11:05	**Łukasz Grabowski**, *Connes-Weiss and Glasner-Weiss...*
11:05-11:15	break
11:15-12:00	**François Le Maître**, *Some global aspects of transitive actions*
12:00-14:00	lunch break
14:00-14:45	**Ruiyuan Chen**, *Étale structures in countable model theory...*
14:45-15:15	coffee break + poster session
15:15-16:00	**Robin Tucker-Drob**, *Measure Equivalence, Schlichting...*
16:00-16:10	break
16:10-16:55	**Konstantin Slutsky**, *Katok's representation theorem...* (zoom)

Wednesday

9:00-9:45	**Felipe García-Ramos**, *The isomorphism relation for Cantor...*
9:45-10:05	coffee break
10:05-10:50	**Su Gao**, *Subshifts of finite symbolic rank*
10:50-11:00	break
11:00-11:45	**Michal Doucha**, *Strong topological Rokhlin property of...*
11:45-11:55	break
11:55-12:40	**Joshua Frisch**, *Characteristic Measures and Minimal...*
.....	
17:45	meeting before the dinner, at Wieczór nad Wisłą
18:00	dinner at Wieczór nad Wisłą

Thursday

9:15-10:00	**Julien Melleray**, *Generic properties in the space of actions...*
10:00-10:20	coffee break
10:20-11:05	**Filippo Calderoni**, *The Borel complexity of the space of...*
11:05-11:15	break
11:15-12:00	**Luca Motto Ros**, *Borel complexity of graph homomorphism*
12:00-14:00	lunch break
14:00-14:45	**Aristotelis Panagiotopoulos**, *The class and dynamics of...*
14:45-15:05	coffee break
15:05-15:50	**Dana Bartošová**, *Maximal factor flows with respect to...*
15:50-16:00	break
16:00-16:45	**Assaf Shani**, *Generic analysis of Borel homomorphisms for...*
16:45-16:55	break
16:55-17:40	**Alexander Kechris**, *The compact action realization problem* (zoom)

Friday

9:00-9:45	**Jan Grebík**, *Derandomization in descriptive graph combinatorics*
9:45-10:05	coffee break
10:05-10:50	**Zoltán Vidnyánszky**, *Complexity of solving Borel linear equations*
10:50-11:00	break
11:00-11:45	**Spencer Unger**, *TBA*
11:45-11:55	break
11:55-12:40	**Stephen Jackson**, *Open and clopen structurings, forcing, and...*

Abstracts

Dana Bartošová, *Maximal factor flows with respect to topologizing a group*

For a Boolean flow of a discrete group G we construct a maximal quotient that is a continuous flow of G with a coarser topology by the technique of near ultrafilters. This was done for the greatest ambit by Koçak and Arvasi and for the universal minimal flow in my thesis. This is work in progress with many open questions.

Filippo Calderoni, *The Borel complexity of the space of left-orderings*

A group is left-orderable if it admits a strict total order that is left-invariant under the group operation. The space of left-orderings of a given countable group is a well studied compact Polish space whose topological and dynamical features interact with the algebraic properties of the group. In this talk I will discuss the Borel complexity of the conjugacy equivalence relation on the spaces of left-orderings. This is joint work with Adam Clay.

Ruiyuan Chen, *Étale structures in countable model theory and descriptive set theory*

An étale structure over a topological space is a "continuous map" from that space to the "space of all structures" in a first-order language. We give an introduction to this fundamental concept from sheaf theory, and show how it can serve as a natural context for generalizations of many results and tools from countable model theory, such as (the usual Polish) spaces of countable structures, Vaught transforms, the Lopez-Escobar theorem, the omitting types theorem, and Scott ranks. If time permits, we will also discuss ongoing work on generalizing these ideas to continuous logic.

Michal Doucha, *Strong topological Rokhlin property of countable groups*

Following Hochman, say that a countable group G has the Strong topological Rokhlin property (STRP) if G admits a generic action on $2^{\mathbb{N}}$, i.e. a continuous action on the Cantor space whose conjugacy class is comeager in the space of all continuous actions of G on $2^{\mathbb{N}}$. I will present a characterization of countable groups with the STRP in symbolic dynamical terms based on certain special class of sofic subshifts. Then I will show a few examples and non-examples of groups with the STRP.
A part of the talk will be devoted to genericity of other dynamical properties in the space of Cantor actions; most notably, to the important and popular notion of shadowing, aka pseudo-orbit tracing property. I will show that, for a countable group G, shadowing is generic in the space of Cantor actions of G if and only if G has the STRP.

Joshua Frisch, *Characteristic Measures and Minimal Subdynamics*

Given a topological dynamical system with G, X a measure on X is said to be characteristic if it is invariant under the automorphism group of the system. In this talk I will give a brief introduction to characteristic measures before addressing the main question, when can you ensure they exist and when can you prove they don't. This is based on joint work with Brandon Seward and Andy Zucker.

Su Gao, *Subshifts of finite symbolic rank*

Inspired by the cutting and stacking process for constructing measure preserving transformations, we define a symbolic rank for topological subshifts that are generated by certain infinite words. In this talk I will report some results concerning subshifts of finite symbolic rank and their topological factors. For example, any minimal Cantor system of finite topological rank is either an odometer or conjugate to a subshift of finite symbolic rank. While an infinite odometer cannot be conjugate to any subshift, there exist subshifts of symbolic rank two whose maximal equicontinuous factor is conjugate to the given odometer. Along the way we will also consider descriptive complexity of various classes of essentially minimal Cantor systems. This is joint work with Ruiwen Li, which is based on some earlier joint work with Ruiwen Li, Cesar Silva, and his students.

Felipe García-Ramos, *The isomorphism relation for Cantor minimal systems and simple amenable groups*

We will talk about Cantor minimal systems and the descriptive complexity of the conjugacy and flip conjugacy relation of these systems. Finally we will mention consequences of the isomorphism relation of amenable simple groups.

Joint work with Deka, Kasprzak, Kunde and Kwietniak.

Łukasz Grabowski, *Connes-Weiss and Glasner-Weiss theorems for Kazhdan equivalence relations, and applications to cost*

This is a report on a joint work with Hector Jardon Sanchez and Samuel Mellick. This work was motivated by the desire to carry out the arguments of Hutchcroft and Pete about cost of Kazhdan groups in the setting of graphings and equivalence relations. In particular, two important ingredients in the arguments of Hutchcroft and Pete are the Connes-Weiss and Glasner-Weiss theorems, and we generalise those theorems to the context of measured equivalence relation. Using those, we show that the "groupoid cost" of a Kazhdan equivalence relation is 1, which generalises the result of Hutchcroft and Pete. In fact, our proof of this last theorem simplifies the arguments of Hutchcroft and Pete also in their original case of countable groups.

Jan Grebík, *Derandomization in descriptive graph combinatorics*

The topic of derandomization is at the center of interest in the theory of distributed algorithms and complexity theory in general. In the LOCAL model of distributed computing, the role of randomness is very well understood. There are examples of classes of graphs and local problems where randomness helps, as well as examples where deterministic and randomized algorithms work equally well.

In descriptive graph combinatorics, an analogous question asks about the information we lose by discarding null sets. There are examples of Borel graphs and local problems that do not admit a Borel measurable solution, but can be solved off of a null set for any Borel probability measure. These results complement, and sometimes also use, the corresponding results from the theory of distributed computing. General derandomization in descriptive graph combinatorics seems to be a challenging problem.

In the first part of the lecture, I will discuss several results of specific local problems where derandomization was achieved. In the second part, I will describe a general derandomization result for local problems on oriented paths. This is a joint work with Vašek Rozhoň.

Stephen Jackson, *Open and clopen structurings, forcing, and hyperaperiodicity*

A number of interesting questions concern the existence of Borel, open, or clopen types of structurings that can be put on equivalence relations. Even in the simple case of free Z^n actions these qustions are interesting and require new techniques. We consider several of these, and use recent methods that involve forcing as well as certain hyperaperiodicity arguments. For example, we will show that there is no open k-lining of free Z^n actions but there is an open less than or equal to k lining. We will consider these and similar questions, some of which are still open.

Alexander Kechris, *The compact action realization problem*

In this talk I will discuss realizations of countable Borel equivalence relations by continuous actions of countable groups, focusing in particular on the problem of realization by continuous actions on compact spaces and more specifically subshifts. This also leads to considering a natural universal space for actions and equivalence relations via subshifts and the study of the descriptive and topological properties in this universal space of various classes of countable Borel equivalence relations, especially the hyperfinite ones.

Krzysztof Krupiński, *Ramsey theory and topological dynamics for first order theories, and an abstract generalization*

In the first part of the talk, I will discuss a part of the theory developed in my joint paper [1] with Junguk Lee and Slavko Moconja which can be viewed as a variant of the celebrated Kechris, Pestov, and Todorčević theory (shortly, KPT theory) [2] in the context of (complete first order) theories. This leads to correspondences between Ramsey-theoretic properties which involve "definable colorings" and dynamical properties of the underlying theory, i.e. properties that are expressed in terms of the action of the group of automorphisms of a monster (i.e. sufficiently saturated and homogeneous) model of the theory in question on the appropriate space of types. On the one hand, this leads to counterparts of some results from KPT theory, but on the other hand, to essentially new results. One of the main contributions are combinatorial criteria for triviality and profiniteness of the Ellis group of the theory in question.

In the second part, I will focus on my ongoing project with Junguk Lee and Slavko Moconja, in which we have adapted the definitions of various definable Ramsey properties from [1] to the general context of 0-dimensional ambits (and some of them even to arbitrary ambits) and we have adapted the proofs of the main results of [1] to this general context, obtaining correspondences between Ramsey theoretic properties (with suitable "definability" requirements on colorings) and dynamical properties of the ambit in question and yielding criteria for triviality and profiniteness of the Ellis group of this ambit.

This general abstract context can be applied to various interesting situations, in particular to:

1. first order theories,
2. definable groups,
3. classical KPT theory.

In (1), one recovers the main results from [KLM]. In (2), it gives us new results and leads to some fundamental questions. A very interesting situation arises in (3) with potential new structural results in this classical context and some natural questions.

References

[1] K. Krupiński, J. Lee, S. Moconja, *Ramsey theory and topological dynamics for first order theories*, Trans. Amer. Math. Soc. 375 (2022), 2553-2596.

[2] A.S. Kechris, V. G. Pestov, S. Todorčević, *Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups*, Geom. Funct. Anal. 15 (2005), 106-189.

Gábor Kun, *When the measurable Hall theorem fails*

Measurable perfect matchings have been intensively studied due to their close relationship to measurable equidecompositions. Most of the theorems use either the expansion of the corresponding measurable graph (Lyons-Nazarov theorem, Banach-Ruziewicz problem) or its hyperfiniteness (measurable and Borel solution to Tarski's circle squaring problem). However, not many examples are known when there is no measurable perfect matching though the measurable Hall condition holds. Most examples admit two-ended components (i.e., of linear growth): in the hyperfinite case we characterized with Bowen and Sabok graphings without measurable perfect matchings via such obstructions.

In this talk I construct for every $d \geq 3$ a d-regular acyclic measurably bipartite graphing that admits no measurable perfect matching. This answers a question of Kechris and Marks. A variant of this construction gives a free pmp action of the free product $(\mathbb{Z}/2\mathbb{Z})^{*d}$ that admits no measurable non-trivial circulation. Another version disproves a conjecture of Peled and Gurel-Gurevich on more dense measurable graphs (called permutons or Markov spaces). I give a probability measure on $[0,1]^2$ whose marginals are equal to the Lebesgue measure on $[0,1]$, all of its sections are atomless but its support does not contain the graph of a measurable perfect matching of $[0,1]$.

Aleksandra Kwiatkowska, *Dendroids via the projective Fraisse limit constructions*

We review the Mohler-Nikiel construction of a universal smooth dendroid. Then we discuss a smooth dendroid obtained from the projective Fraisse limit construction, which shares some similarities with the Mohler-Nikiel one, nevertheless it is not homeomorphic to it. This is joint work in progress with W. Charatonik and R. Roe.

François Le Maître, *Some global aspects of transitive actions*

We will investigate the space of all transitive actions on the integers of a fixed countable group, with an emphasis on groups acting on trees and high transitivity. This is based on joint work with Fima, Moon, Stalder and Carderi, Gaboriau, Stalder.

Julien Melleray, *Generic properties in the space of actions of non finitely generated groups on the Cantor space*

Following recent work of M. Doucha, we investigate generic properties in the space of actions of a given countable group on the Cantor space. In this talk I will focus on the case of non finitely generated groups, showing how some properties of the group (such as amenability or local finiteness) are reflected by generic properties in the space of actions. As a consequence of this (and Doucha's) work I will explain why all conjugacy classes in the space of actions are meager when the acting group is countable, amenable and not finitely generated (whether the same property holds true for all non finitely generated groups is an open question).

This is part of ongoing joint work with M. Doucha and T. Tsankov.

Luca Motto Ros, *Borel complexity of graph homomorphism*

We consider several classes of countable graphs naturally arising in the context of graph theory and combinatorics, and analyze the complexity with respect to Borel reducibility of the homomorphism relation on them. The outcome is a sort of empirical dichotomy asserting that for each of these classes, either there are few incomparable graphs, or else the homomorphism relation is invariantly universal (and hence complete for analytic quasi-orders). This considerably extends a result by Louveau and Rosendal and shows that many widely considered graph-theoretic constraints do not affect the complexity of graph homomorphism. (Joint work with Salvatore Scamperti)

Aristotelis Panagiotopoulos, *The class and dynamics of α-balanced Polish groups*

A Polish group is TSI if it admits a two-side invariant metric. It is CLI if it admits complete and left-invariant metric. The class of CLI groups contains every TSI group but there are many CLI groups that fail to be TSI. In this talk we will introduce the class of α-balanced Polish groups where α ranges over all countable ordinals. We will show that these classes completely stratify the space between TSI and CLI. We will also introduce "generic α-unbalancedness": a turbulence-like obstruction to classification by actions of α-balanced Polish groups. Finally, for each α we will provide an action of an α-balanced Polish group whose orbit equivalence relation is not classifiable by actions of any β-balanced Polish group with $\beta < \alpha$. This is joint work with Shaun Allison.

Oleg Pikhurko, *Circle Squaring with Pieces of Small Boundary and Low Borel Complexity*

Tarski's Circle Squaring Problem from 1925 asks whether it is possible to partition a disk in the plane into finitely many pieces and reassemble them via isometries to yield a partition of a square of the same area. It was finally resolved by Laczkovich in 1990 in the affirmative. Recently, several new proofs have emerged which achieve circle squaring with better structured pieces: namely, pieces which are Lebesgue measurable and have the property of Baire (Grabowski-Mathe-Pikhurko) or even are Borel (Marks-Unger).

I will discuss our result with Andras Mathe and Jon Noel that circle squaring is possible with Borel pieces whose boundaries have upper Minkowski dimension less than 2 (in particular, each piece is Jordan measurable). We also improve the Borel complexity of the pieces: namely, we show that each piece can be taken to be a Boolean combination of F_σ sets.

Assaf Shani, *Generic analysis of Borel homomorphisms for the finite Friedman-Stanley jumps*

This talk concerns the Friedman-Stanley jumps $=^{+n}$, for $n = 1, 2, ...$ and $n = \omega$. Specifically, we consider the problem of constructing a Borel reduction from $=^{+n}$ to some other equivalence relation. For $n = 1$ the situation is well understood and there are many such results. We present a technique for finding such a reduction, when $n > 1$, based on Baire-category methods. As corollaries, we conclude that $=^{+\omega}$ is prime (as defined by Clemens), and that $=^{+n}$ is in the spectrum of the meager idea (as defined by Kanovei, Sabok, and Zapletal).

Konstantin Slutsky, *Katok's representation theorem for multidimensional Borel flows*

Study of $\mathbb{R}$-flows in both ergodic theory and Borel dynamics often relies on the suspension flow construction, which connects flows with $\mathbb{Z}$-actions. This connection, for instance, is essential in the theory of Kakutani equivalence.

The construction of a suspension flow does not readily generalize to higher dimensional flows. Within the framework of ergodic theory, Katok was able to overcome this obstacle by introducing the so-called special flows, which are build over $\mathbb{Z}^d$-actions and $\mathbb{R}^d$-valued cocycles. His representation theorem asserts that any ergodic measure-preserving flow arises as a special flow over some measure-preserving $\mathbb{Z}^d$-action and a cocycle, which can be arbitrarily close to an isometry. This opened gates for the multidimensional theory of Kakutani equivalence, pioneered by del Junco and Rudolph.

In Borel dynamics, $\mathbb{R}$-flows have been studied quite actively in the recent years, and many of the fundamental ergodic theoretical results were showed to hold in this context as well. Much less is known about multidimensional Borel flows and their relation to $\mathbb{Z}^d$-actions, in particular.

In this talk, we present a Borel version of Katok's representation theorem for $\mathbb{R}^d$-flows. Our main tool will be the framework of partial actions for constructing orbit equivalent actions of Polish groups. While related ideas have been employed in ergodic theory and Borel dynamics for many years, the particular viewpoint of partial actions simplifies construction of orbit equivalent actions of distinct groups. And we therefore hope that it might find further applications.

Sławomir Solecki, *Descriptive Set Theory and generic measure preserving transformations*

One of the areas of interest of Descriptive Set Theory is dynamics of Polish groups, that is, groups carrying a group topology that is separable and completely metrizable. Such groups, like for example, the unitary group of the infinite dimensional Hilbert space or the homeomorphism group of the unit interval, are not, in general, locally compact. Therefore, in studying their dynamics, classical methods relying on Haar measure are not available. They can often be replaced by descriptive set theoretic or combinatorial tools.

I will describe how the descriptive set theoretic point of view led to a recent answer to an old question in Ergodic Theory. The question lies within a long-established theme, going back to the work of Halmos and Rokhlin, of investigating generic measure preserving transformations. The answer to the question rests on an analysis of unitary representations of a certain non-locally compact Polish group that can be viewed as an infinite dimensional torus.

Ádám Timár, *Poisson matchings of optimal tail via matchings in graphings*

Consider the following purely probabilistic problem. Take two infinite random discrete sets of points in the Euclidean space whose distributions are invariant under isometries. Find a "factor" perfect matching between the two, where factor means, intuitively, that every point can determine its pair using local information and using the same method. We want to make the probability that some fixed point is at distance at least r from its pair decay as fast as possible. A recent result of Bowen, Kun, and Sabok has become an important tool in settling this question for Poisson point processes, where we found a construction with optimal tail, significantly improving on previous ones.

Todor Tsankov, *Maximal highly proximal flows of locally compact groups*

The notion of a highly proximal extension of a flow generalizes the one of an almost one-to-one extension (injective on a dense G_δ set), which is an important tool in topological dynamics. The existence of maximal such extensions was proved by Auslander and Glasner in the 70s for minimal flows using an abstract argument, and a concrete construction using near-ultrafilters was recently given by Zucker for arbitrary flows. When the acting group is discrete, the MHP extension is nothing but the Stone space of the Boolean algebra of the regular open sets of the space. We give yet another construction of the MHP extension for arbitrary topological groups and prove that for MHP flows of a locally compact group G, the stabilizer map $x \mapsto G_x$ is continuous (for general flows, this map is only semi-continuous). This is a common generalization of a theorem of Frolík that the set of fixed points of a homeomorphism of a compact, extremally disconnected space is open and a theorem of Veech that the action of a locally compact group on its greatest ambit is free. This is joint work with Adrien Le Boudec.

Robin Tucker-Drob, *Measure Equivalence, Schlichting Completions, and Baumslag-Solitar groups*

A subgroup H of a group Γ is commensurated by Γ if all H-orbits in Γ/H are finite. In this situation, the closure of Γ in the group of all permutations of Γ/H is a totally disconnected locally compact group called the Schlichting completion of the pair (Γ, H) and we denote it $G(\Gamma, H)$. We show that if H and K are amenable commensurated subgroups of Γ and Λ respectively such that the associated Schlichting completions $G(\Gamma, H)$ and $G(\Lambda, K)$ are both hyperbolic with trivial amenable radical, then every measure equivalence coupling of Γ with Λ descends canonically to a measure equivalence coupling of the (possibly nonunimodular) groups $G(\Gamma, H)$ and $G(\Lambda, K)$. This unifies and generalizes theorems of Houdayer–Raum, Kida, and Monod–Shalom. I will discuss the consequences of this for the open problem of classifying Baumslag-Solitar groups up to measure equivalence.

Spencer Unger, *TBA*

Zoltán Vidnyánszky, *Complexity of solving Borel linear equations*

In this talk, I will discuss infinitary generalizations of the Bulatov-Zhuk Dichotomy theorem, which states that a homomorphism problem is either easy (in P) or hard (NP-complete). In the Borel context a different picture emerges, namely, it is already hard to decide solvability of systems of linear equations. The main focus of the talk will be the proof of the latter statement.

Andrew Zucker, *Ultracoproducts of G-flows*

Given a topological group G, a G-flow is a continuous action of G on a compact Hausdorff space X. This talk will discuss a notion of ultracoproduct for G-flows, which arise from considering ultraproducts of commutative G-C^*-algebras by Gelfand duality. We apply the construction to develop an understanding of the properties of various classes of subflows of a flow, i.e. minimal, topologically transitive, etc. For groups which are locally Roelcke precompact, ultracoproducts of G-flows lead to a well-behaved notion of weak containment for a wide class of G-flows, and in particular for all G-flows when G is locally compact. In ongoing joint work with Gianluca Basso, we apply ultracoproducts of G-flows to achieve a new characterization of those Polish groups G with the property that every minimal flow has a comeager orbit.